

**~Test 2: Vectors, Circles & Relative(s)**

Equations and Constants:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

$$\bar{v} = \frac{1}{2}(v_i + v_f)$$

$$|g| = 10 \text{ m/s}^2$$

$$x = \frac{1}{2}at^2 + v_i t + x_i$$

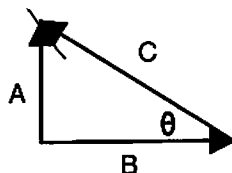
$$v = at + v_i$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$a_c = \frac{v^2}{r}$$

2011 (online)

82 total

**Multiple Choice: Choose the letter of the best answer. 3 points each.**

1. C or D Three vectors are shown in the diagram above. Which of the following expressions would match the diagram?
- a.  $A + B = C$     b.  $A + C = B$     c.  $C + B = A$     d.  $A - B = C$     e.  $C - B = A$

Questions 2 and 3 refer to the following vectors:

$$\vec{A} = 5\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{B} = -4\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{C} = 4\hat{i} + 5\hat{j} - 3\hat{k}$$

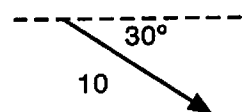
2. A What is  $B - A$ ?
- a.  $-9\hat{i} + \hat{j} - \hat{k}$     b.  $-9\hat{i} - 5\hat{j} + 7\hat{k}$     c.  $-\hat{i} - 5\hat{j} + 7\hat{k}$     d.  $\hat{i} + \hat{j} - \hat{k}$
3. B Which two vectors have the same magnitude?
- a. A & B.    b. A & C.    c. B & C.    d. none, they are all different.
4. E The initial velocity of an object is  $2\hat{i} + 3\hat{j}$  m/s. It undergoes a constant acceleration of  $-\hat{i} + 2\hat{j}$  m/s<sup>2</sup> for 4 seconds. What is its final velocity?
- a.  $\hat{i} + 5\hat{j}$  m/s    b.  $3\hat{i} + \hat{j}$  m/s    c.  $7\hat{i} + 14\hat{j}$  m/s    d.  $-6\hat{i} - 5\hat{j}$  m/s    e.  $-2\hat{i} + 11\hat{j}$  m/s
5. B If a moving object has an acceleration that is always perpendicular to its velocity, what must be happening?
- a. It must always be slowing down.  
b. It must have a constant speed.  
c. Its path must be a parabola.  
d. It must slow down, stop and then speed up in the opposite direction.  
e. What? It is clearly impossible for this situation to exist.

6. A Vectors  $r_i$  and  $r_f$  are shown above. Which of the following would best represent  $\Delta r$ ?



7. A What is the vector in unit-vector notation for the vector shown in the diagram to the right?

- a.  $8.7\hat{i} - 5\hat{j}$     b.  $8.7\hat{i} + 5\hat{j}$     c.  $5\hat{i} + 8.7\hat{j}$   
d.  $1.5\hat{i} - 9.9\hat{j}$     e.  $1.5\hat{i} - 9.9\hat{j}$



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Questions 8 to 10 refer to the following:

Charlie is riding a Ferris Wheel of radius 7 meters. He is moving with a constant speed of 3 m/s.

8. B What is the magnitude of his acceleration when he is at his lowest point?  
 a. 16.3 m/s<sup>2</sup>.    b. 1.3 m/s<sup>2</sup>.    c. 0.8 m/s<sup>2</sup>.    d. 8.7 m/s<sup>2</sup>.    e. 6.3 m/s<sup>2</sup>.
9. C When he is at the lowest point on the ride, which of the following vectors best represent his acceleration?  
 a.    b.    c.    d.    e. none of these.
10. B With how many "rpm"s is he rotating?  
 a. 0.068 rpm.    b. 4.09 rpm.    c. 14.7 rpm.    d. 43.9 rpm.    e. 180 rpm.

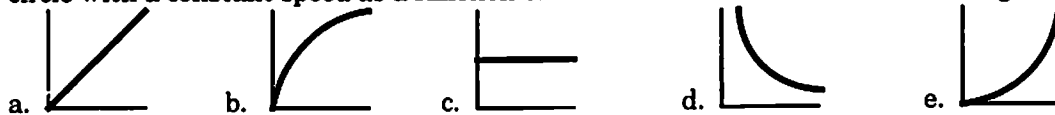
Questions 11 to 13 refer to the following:

A river is 100 m wide and has a current that is flowing at 6 m/s. A duck has a water speed of 8 m/s.

100 m



11. B What would be the minimum time the duck needs to cross the river?  
 a. 16.7 s.    b. 12.5 s.    c. 10.0 s.    d. 7.14 s.
12. D What would be the fastest possible speed of the duck with respect to the ground?  
 a. 5.3 m/s.    b. 8 m/s.    c. 10 m/s.    d. 14 m/s.
13. A If the duck had a water velocity directed due west, what would be the velocity of the duck with respect to the ground?  
 a.  $-8\mathbf{i} - 6\mathbf{j}$  m/s.    b.  $8\mathbf{i} + 6\mathbf{j}$  m/s.    c.  $8\mathbf{i} - 10\mathbf{j}$  m/s.    d.  $-10\mathbf{i} - 8\mathbf{j}$  m/s.
14. D Which of the following would best represent the acceleration of an object traveling in a circle with a constant speed as a function of the radius of the circle it was moving in?

**Problem Solving: Show all work. 10 points each.**

15. The position in meters as a function of time in seconds for an object is given by

$$\vec{r} = (-t^3 + t^2 + 5)\hat{i} + (-5t^2 + 15t)\hat{j}$$

What is the average velocity of the object for the first 4 seconds?

*-6 if v(4)*

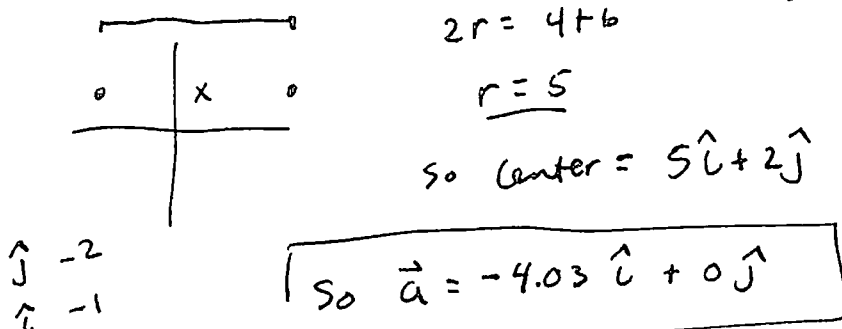
$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{[(-4^3 + 4^2 + 5)\hat{i} + (-5(4)^2 + 15(4))\hat{j}] - [5\hat{i} + 0\hat{j}]}{4}$$

$$= \frac{(-43\hat{i} - 20\hat{j}) - (5\hat{i} + 0\hat{j})}{4} = \frac{-48\hat{i} - 20\hat{j}}{4}$$

$$\boxed{\bar{\vec{v}} = -12\hat{i} - 5\hat{j} \text{ m/s}}$$

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16. Some students do a lab recording the position of an object that is rotating with a constant speed. They notice that at one time, the position of the object is  $-4\hat{i} + 2\hat{j}$ . Then 3.5 seconds later the, object has made exactly half a rotation and the position is  $6\hat{i} + 2\hat{j}$ . In unit-vector notation, what is the acceleration of the object when it is at the second position?



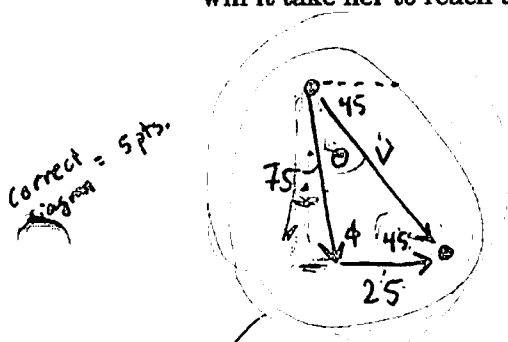
$$V = \frac{2\pi r}{T} = \frac{2\pi(5)}{2(3.5)} = \frac{10\pi}{7}$$

$$V = 4.49 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{(4.49)^2}{5}$$

$$a_c = 4.03 \text{ m/s}^2$$

17. A plane is flying with an airspeed of 75 km/h. The pilot wants to go straight to an airport that is 300 km SE of her current position. Ground Control reports that there is a constant wind velocity of 25 km/h E. She orients the plane in such a way that she flies directly to the airport. How long will it take her to reach the airport?



$$V = V_{PA} + V_{AG}$$

$$\therefore 300 = (90.6)t$$

$$\text{or } 75^2 = 25^2 + v^2 - 2(25)v \cos 45$$

$$5000 = v^2 - 35.4v$$

$$0 = v^2 - 35.4v - 5000$$

$$v = \frac{35.4 \pm \sqrt{(35.4)^2 - 4(1)(-5000)}}{2}$$

$$v = \frac{35.4 \pm 146}{2} = 90.6$$

$$t = 3.31 \text{ hours}$$

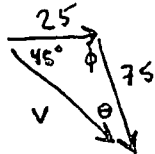
$$\frac{\sin 45}{75} = \frac{\sin \theta}{25}$$

$$\theta = 13.63^\circ \rightarrow \therefore \phi = 180 - 13.6 - 45 = 121.4^\circ$$

$$V = 90.6$$

$$\frac{\sin 121.4}{V} = \frac{\sin 45}{75}$$

18. The earth has a radius of 6400 km. How long would one day be if the earth rotated so fast that the centripetal acceleration of someone standing on the equator was equal to  $9.8 \text{ m/s}^2$ ? (Also, in this situation, what would be the centripetal acceleration for someone at the North Pole?)



$$\therefore r = 6.4 \times 10^6 \text{ m}$$

$$9.8 = \frac{v^2}{r}$$

$$v^2 = (9.8)(6.4 \times 10^6)$$

$$v = 7920 \text{ m/s}$$

$$V = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{V}$$

$$T = 5078 \text{ sec.}$$

$$(= 1.41 \text{ hr} = 84.6 \text{ min})$$

$$\text{@ North Pole, } v=0, \therefore a_c=0!$$

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